Spiral plat avec une seule courbe terminale externe

Influence d'une déformation de la courbe terminale

Calculs numériques et approximations de Haag

Caractéristiques du spiral

Référence :C:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d_V := 1.1\cdot mm \quad d_B := d1_{sp} \quad p_{sp} = 0.135\,mm \quad n_{sp} := \dfrac{d2_{sp} - d_B}{2\cdot p_{sp}}$

$L := \pi\cdot\dfrac{n_{sp}}{2}\cdot\left(d2_{sp} + d_B\right) \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot\left(\alpha - \alpha_A\right) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A^{\,2} - r_s(\alpha)^2\right) \quad s(\alpha) := r_A\cdot\left(\alpha - \alpha_A\right) - \dfrac{a}{2}\cdot\left(\alpha - \alpha_A\right)^2$

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Courbe terminale externe

$r_{t1} := 0.8 \quad r_{Ph} := racine\left[\left(2\cdot r_{t1} - 1\right)^4 - 4\cdot\left(1 - r_{t1}\right)^4 - \pi^2\cdot r_{t1}^{\,2}\cdot\left(1 - r_{t1}\right)^2, r_{t1}\right]\cdot r_A \quad r_{Ph} = 0.832\,r_A$

$r_{t2}\left(r_{t1}\right) := 2\cdot r_{t1} - r_A \quad \beta_0 := arctan\left[\dfrac{\pi\cdot r_{Ph}}{2\cdot\left(r_A - r_{Ph}\right)}\right] \quad \beta_0 = 82.695\,deg \quad l_t\left(r_{t1}\right) := r_{t2}\left(r_{t1}\right)\cdot\beta_0 + \pi\cdot r_{t1}$

$X_{0t1}\left(r_{t1}, \alpha_t\right) := r_A - r_{t1} + r_{t1}\cdot cos\left(\alpha_t\right) \quad Y_{0t1}\left(r_{t1}, \alpha_t\right) := r_{t1}\cdot sin\left(\alpha_t\right)$

$X_{0t2}\left(r_{t1}, \beta_t\right) := -r_{t2}\left(r_{t1}\right)\cdot cos\left(\beta_t\right) \quad Y_{0t2}\left(r_{t1}, \beta_t\right) := -r_{t2}\left(r_{t1}\right)\cdot sin\left(\beta_t\right) \quad \mathbf{OA} := r_A\cdot e^{i\cdot\pi}$

Position du point d'attache $\quad \alpha_B := mod\left(\psi_0 + \pi, 2\cdot\pi\right) \quad \alpha_B = 60\,deg \quad r_B := 0.5\cdot d_B$

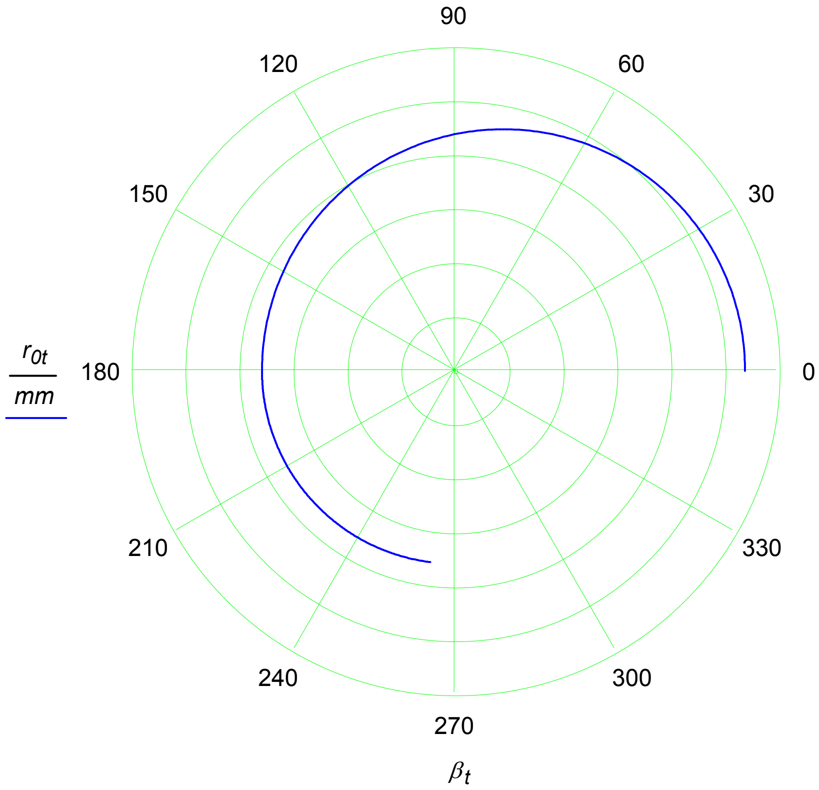
$\mathbf{OB} := r_B\cdot e^{i\cdot\left(\pi+\psi_0\right)} \quad L_t\left(r_{t1}\right) := l_t\left(r_{t1}\right) + L$

Graphe

$n_t := 201 \quad j := 0 .. n_t - 1 \quad \Delta\alpha_t := \dfrac{\pi}{n_t - 1} \quad \alpha_{t_j} := j\cdot\Delta\alpha_t \quad X_{t_j} := X_{0t1}\left(r_{Ph}, \alpha_{t_j}\right) \quad Y_{t_j} := Y_{0t1}\left(r_{Ph}, \alpha_{t_j}\right)$

$\Delta\beta_t := \dfrac{\beta_0}{n_t - 1} \quad \beta_{t_j} := j\cdot\Delta\beta_t \quad X_{t2_j} := X_{0t2}\left(r_{Ph}, \beta_{t_j}\right) \quad Y_{t2_j} := Y_{0t2}\left(r_{Ph}, \beta_{t_j}\right)$

$X_t := pile\left(X_t, X_{t2}\right) \quad Y_t := pile\left(Y_t, Y_{t2}\right) \quad r_{0t} := \overrightarrow{\sqrt{X_t^{\,2} + Y_t^{\,2}}} \quad \beta_t := \overrightarrow{Atan\left(X_t, Y_t\right)}$



Perturbation de période en position horizontale

Cas d'une courbe de Phillips

Paramètres de la courbe terminale externe

$Z_{0t1}(r_{t1}, \alpha) := X_{0t1}(r_{t1}, \alpha) + i \cdot Y_{0t1}(r_{t1}, \alpha)$ $\quad$ $Z_{0t2}(r_{t1}, \alpha) := X_{0t2}(r_{t1}, \alpha) + i \cdot Y_{0t2}(r_{t1}, \alpha)$

$$Z_1(r_{t1}) := \frac{1}{r_A^2} \cdot \left(\int_0^{\pi} Z_{0t1}(r_{t1}, \alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} Z_{0t2}(r_{t1}, \beta) \cdot r_{t2}(r_{t1}) \, d\beta \right) - i$$

$\rho_{01} := |Z_1(r_{Ph})|$ $\quad$ $\varphi_{01} := arg(Z_1(r_{Ph}))$ $\quad$ $\boxed{\rho_{01} = 0}$ $\quad$ $\boxed{\varphi_{01} = -90\,deg}$

$$Z_2(r_{t1}) := \frac{1}{r_A^3} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot Z_{0t1}(r_{t1}, \alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2}(r_{t1}) \cdot \beta) \cdot Z_{0t2}(r_{t1}, \beta) \cdot r_{t2}(r_{t1}) \, d\beta \right] + 1$$

$\rho_{02} := |Z_2(r_{Ph})|$ $\quad$ $\varphi_{02} := arg(Z_2(r_{Ph}))$ $\quad$ $\boxed{\rho_{02} = 1.055}$ $\quad$ $\boxed{\varphi_{02} = 147.579\,deg}$

Modifications de la forme de la courbe terminale

Valeurs de test $\quad$ $x_1 := 1.02$ $\quad$ $x_2 := 0.98$ $\quad$ $r_{t1} := x_1 \cdot r_{Ph}$

$\rho_1(r_{t1}) := |Z_1(r_{t1})|$ $\quad$ $\varphi_1(r_{t1}) := arg(Z_1(r_{t1}))$ $\quad$ $\rho_2(r_{t1}) := |Z_2(r_{t1})|$ $\quad$ $\varphi_2(r_{t1}) := arg(Z_2(r_{t1}))$

$\boxed{\rho_1(r_{t1}) = 0.082}$ $\quad$ $\boxed{\varphi_1(r_{t1}) = 168.521\,deg}$ $\quad$ $\boxed{\rho_2(r_{t1}) = 1.274}$ $\quad$ $\boxed{\varphi_2(r_{t1}) = 156.245\,deg}$

Calcul numérique de la perturbation de période

$$\mathbf{w_A}(r_{t1},\theta) := \left[i\cdot\left(r_A\cdot\rho_1(r_{t1})\cdot e^{-i\cdot\varphi_1(r_{t1})} + 2\cdot a\right) + \frac{\theta}{L_t(r_{t1})}\cdot r_A^{\,2}\cdot\rho_2(r_{t1})\cdot e^{-i\cdot\varphi_2(r_{t1})} \right]\cdot exp\left(i\cdot\theta\cdot\frac{l_t(r_{t1})}{L_t(r_{t1})}\right)\cdot \boldsymbol{OA}$$

$$\mathbf{w_B}(r_{t1},\theta) := \left[-i\cdot(i\cdot r_B + 2\cdot a) - \frac{\theta}{L_t(r_{t1})}\cdot r_B^{\,2} \right]\cdot exp\left(i\cdot\theta\cdot\frac{l_t(r_{t1}) + L}{L_t(r_{t1})}\right)\cdot \boldsymbol{OB}$$

$$\mathbf{w}(r_{t1},\theta) := \frac{\theta}{L_t(r_{t1})}\cdot\left(\mathbf{w_A}(r_{t1},\theta) + \mathbf{w_B}(r_{t1},\theta)\right) \qquad \mathbf{w}(r_{t1},\theta_0) = 0.019 + 0.019i\, mm$$

$$\sigma 2 := \frac{r_A^{\,2} + r_B^{\,2}}{2} \qquad \sigma 2 = 2.705\, mm^2 \qquad X_w(r_{t1},\theta) := \frac{\left(\left|\mathbf{w}(r_{t1},\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_w(r_{t1},\theta) := \frac{d}{d\theta}X_w(r_{t1},\theta)$$

$$\delta_H(r_{t1},\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi} \gamma_w(r_{t1},\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\, d\varphi$$

$$\mu_H(x,\theta_0) := -86400\cdot\delta_H(x\cdot r_{Ph},\theta_0) \qquad \boxed{\mu_H(x_1,\theta_0) = 2.401} \qquad \boxed{\mu_H(x_1, 180\cdot deg) = 0.655}$$

Solution analytique

$$A(r_{t1}) := \frac{1}{L_t(r_{t1})^2}\cdot\left[r_A^{\,4}\cdot\rho_1(r_{t1})^2 + r_B^{\,4} + 4\cdot a\cdot r_A^{\,3}\cdot\rho_1(r_{t1})\cdot cos(\varphi_1(r_{t1})) + 4\cdot a^2\cdot\left(r_A^{\,2} + r_B^{\,2}\right)\right]$$

$$B(r_{t1}) := \frac{3}{2\cdot L_t(r_{t1})^4}\cdot\left(r_A^{\,6}\cdot\rho_2(r_{t1})^2 + r_B^{\,6}\right)$$

$$C1(r_{t1}) := 2\cdot a\cdot r_A\cdot r_B\cdot\left(r_B\cdot sin(\psi_0) - r_A\cdot\rho_1(r_{t1})\cdot cos(\psi_0 + \varphi_1(r_{t1})) - 2\cdot a\cdot cos(\psi_0)\right)$$

$$C(r_{t1}) := \frac{2}{L_t(r_{t1})^2}\cdot\left(C1(r_{t1}) + r_A^{\,2}\cdot r_B^{\,2}\cdot\rho_1(r_{t1})\cdot sin(\psi_0 + \varphi_1(r_{t1}))\right)$$

$$D1(r_{t1}) := r_A\cdot r_B^{\,2}\cdot\rho_1(r_{t1})\cdot cos(\psi_0 + \varphi_1(r_{t1})) + r_A^{\,2}\cdot r_B\cdot\rho_2(r_{t1})\cdot sin(\psi_0 + \varphi_2(r_{t1}))$$

$$D(r_{t1}) := \frac{2\cdot r_A\cdot r_B}{L_t(r_{t1})^3}\cdot\left[D1(r_{t1}) + 2\cdot a\cdot\left(r_B^{\,2}\cdot cos(\psi_0) - r_A^{\,2}\cdot\rho_2(r_{t1})\cdot cos(\psi_0 + \varphi_2(r_{t1}))\right)\right]$$

$$K(r_{t1}) := \frac{2}{L_t(r_{t1})^4}\cdot r_A^{\,3}\cdot r_B^{\,3}\cdot\rho_2(r_{t1})\cdot cos(\psi_0 + \varphi_2(r_{t1}))$$

$$F(x) := J0(x) - x\cdot J1(x) \qquad H(x) := x\cdot\left(1 + x^2\right)\cdot J1(x) - 2\cdot x^2\cdot J0(x) \qquad G(x) := -x\cdot(J1(x) + x\cdot J0(x))$$

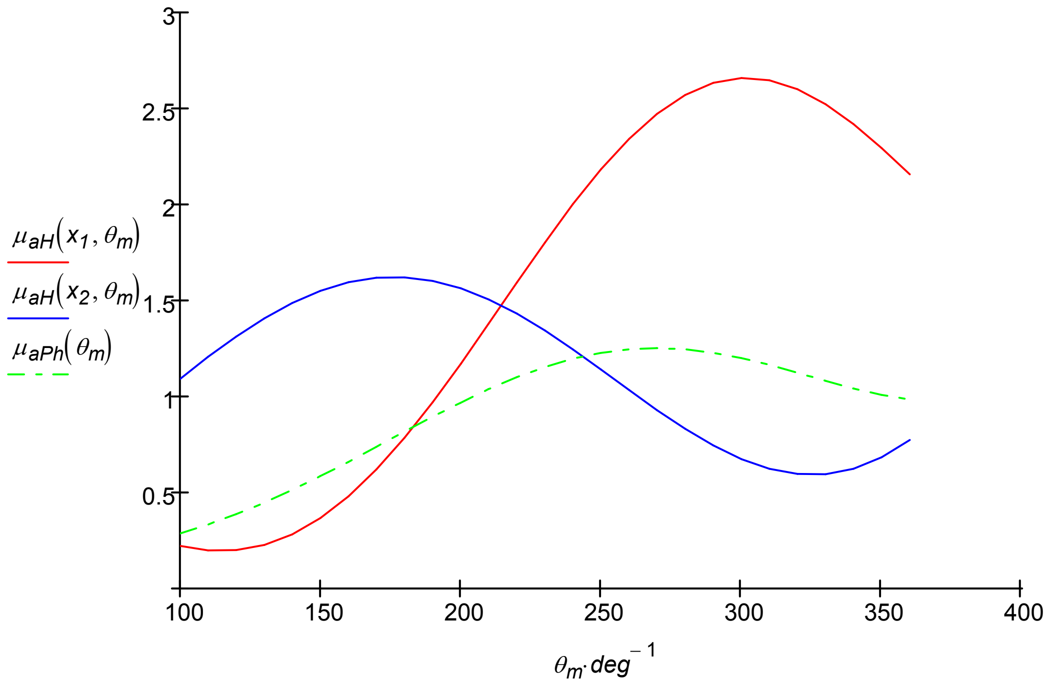
$$\delta_{aH}(r_{t1},\theta_0) := \frac{-1}{\sigma 2}\cdot\left(A(r_{t1}) + B(r_{t1})\cdot\theta_0^{\,2} + C(r_{t1})\cdot F(\theta_0) + D(r_{t1})\cdot G(\theta_0) + K(r_{t1})\cdot H(\theta_0)\right)$$

$$\mu_{aH}(x,\theta_0) := -86400\cdot\delta_{aH}(x\cdot r_{Ph},\theta_0) \qquad \boxed{\mu_{aH}(x_1,\theta_0) = 2.472} \qquad \boxed{\mu_{aH}(x_1, 180\cdot deg) = 0.784}$$

Courbes Phillips

$$\mu_{aPh}(\theta_0) := -86400\cdot\delta_{aH}(r_{Ph},\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = 1.251} \qquad \boxed{\mu_{aPh}(180\cdot deg) = 0.816}$$

$\theta_m := 100 \cdot deg, 110 \cdot deg .. 360 \cdot deg$ $\qquad x_1 := 1.02 \qquad x_2 := 0.98$



Déplacement du centre de gravité

Modifications de la forme des courbes terminales

$\kappa := 0.367$

$$f_g(r_{t1}, \theta, s) := \left[1 + i \cdot \theta \cdot \left(\frac{s}{L_t(r_{t1})} - \kappa\right)\right] \cdot e^{i \cdot \theta \cdot \frac{s}{L_t(r_{t1})}} \qquad f_{g1}(r_{t1}, \theta, s) := \frac{d}{ds} f_g(r_{t1}, \theta, s)$$

$$f_{gA}(r_{t1}, \theta) := f_g(r_{t1}, \theta, l_t(r_{t1})) \qquad f_{gB}(r_{t1}, \theta) := f_g(r_{t1}, \theta, l_t(r_{t1}) + L)$$

$$f_{g1A}(r_{t1}, \theta) := f_{g1}(r_{t1}, \theta, l_t(r_{t1})) \qquad f_{g1B}(r_{t1}, \theta) := f_{g1}(r_{t1}, \theta, l_t(r_{t1}) + L)$$

$$\zeta_{at}(r_{t1}, \theta) := \left[\left(r_A \cdot \rho_1(r_{t1}) \cdot e^{-i \cdot \varphi_1(r_{t1})} + 2 \cdot a\right) \cdot f_{gA}(r_{t1}, \theta) - r_A{}^2 \cdot \rho_2(r_{t1}) \cdot e^{-i \cdot \varphi_2(r_{t1})} \cdot f_{g1A}(r_{t1}, \theta)\right] \cdot \mathbf{OA}$$

$$\zeta_{at'}(r_{t1}, \theta) := \left[-(i \cdot r_B + 2 \cdot a) \cdot f_{gB}(r_{t1}, \theta) + r_B{}^2 \cdot f_{g1B}(r_{t1}, \theta)\right] \cdot \mathbf{OB}$$

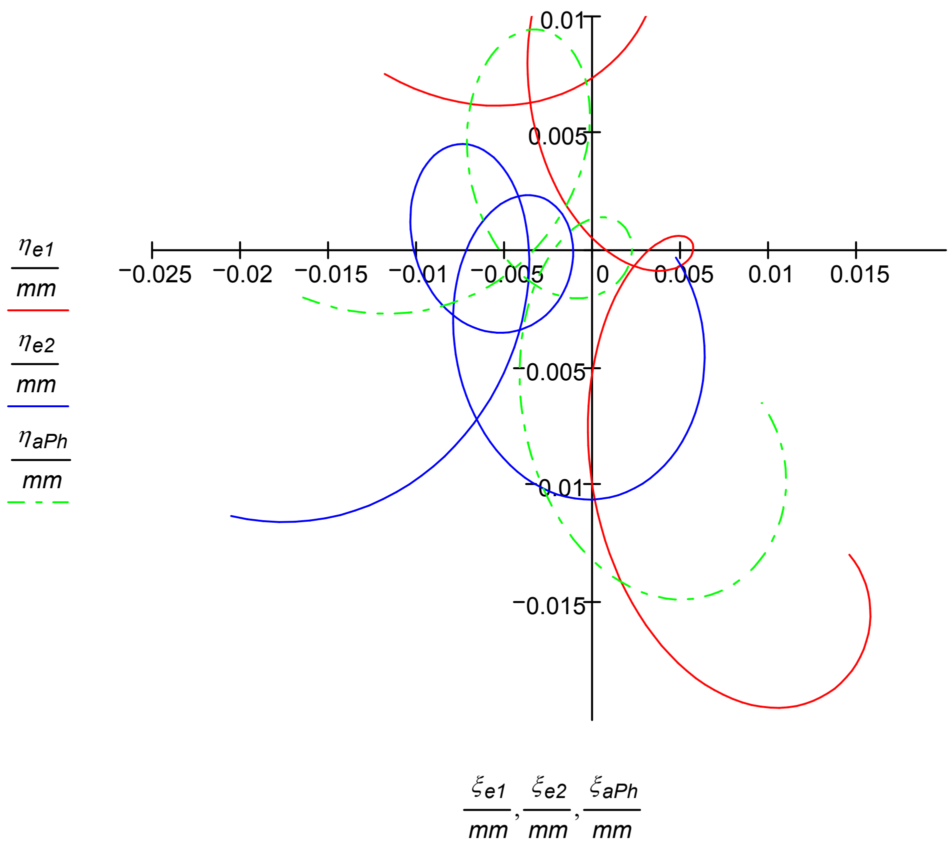
$$\zeta_a(x, \theta) := \frac{1}{L_t(x \cdot r_{Ph})} \cdot \left(\zeta_{at}(x \cdot r_{Ph}, \theta) + \zeta_{at'}(x \cdot r_{Ph}, \theta)\right)$$

Courbes Phillips $\qquad \zeta_{aPh}(\theta) := \zeta_a(1, \theta)$

Graphes du déplacement du centre de gravité

$$n := 201 \qquad i := 0 .. n-1 \qquad \Delta\theta := \frac{4 \cdot \pi}{n-1} \qquad \theta_i := -2 \cdot \pi + i \cdot \Delta\theta \qquad \xi_{aPh_i} := Re(\zeta_{aPh}(\theta_i)) \qquad \eta_{aPh_i} := Im(\zeta_{aPh}(\theta_i))$$

$$\xi_{e1_i} := Re(\zeta_a(x_1, \theta_i)) \qquad \eta_{e1_i} := Im(\zeta_a(x_1, \theta_i)) \qquad \xi_{e2_i} := Re(\zeta_a(x_2, \theta_i)) \qquad \eta_{e2_i} := Im(\zeta_a(x_2, \theta_i))$$



Perturbation de période en position verticale

Modifications de la forme des courbes terminales

$$f(r_{t1}, \theta_0, s) := \frac{s}{L_t(r_{t1})} \cdot \left[\left(\kappa - \frac{s}{L_t(r_{t1})} \right) \cdot J0\left(\theta_0 \cdot \frac{s}{L_t(r_{t1})} \right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s}{L_t(r_{t1})} \right) \right]$$

$$f_1(r_{t1}, \theta_0, s) := \frac{d}{ds} f(r_{t1}, \theta_0, s)$$

$$f_A(r_{t1}, \theta_0) := f(r_{t1}, \theta_0, l_t(r_{t1})) \qquad f_B(r_{t1}, \theta_0) := f(r_{t1}, \theta_0, l_t(r_{t1}) + L)$$

$$f_{1A}(r_{t1}, \theta_0) := f_1(r_{t1}, \theta_0, l_t(r_{t1})) \qquad f_{1B}(r_{t1}, \theta_0) := f_1(r_{t1}, \theta_0, l_t(r_{t1}) + L)$$

$$Z_{a1}(r_{t1}, \theta_0) := \left[\left(r_A \cdot \rho_1(r_{t1}) \cdot e^{-i \cdot \varphi_1(r_{t1})} + 2 \cdot a \right) \cdot f_A(r_{t1}, \theta_0) - r_A^{\,2} \cdot \rho_2(r_{t1}) \cdot e^{-i \cdot \varphi_2(r_{t1})} \cdot f_{1A}(r_{t1}, \theta_0) \right] \cdot \boldsymbol{OA}$$

$$Z_{a2}(r_{t1}, \theta_0) := \left[-(i \cdot r_B + 2 \cdot a) \cdot f_B(r_{t1}, \theta_0) + r_B^{\,2} \cdot f_{1B}(r_{t1}, \theta_0) \right] \cdot \boldsymbol{OB}$$

$$Z_a(r_{t1}, \theta_0) := \frac{1}{L_t(r_{t1})} \cdot \left(Z_{a1}(r_{t1}, \theta_0) + Z_{a2}(r_{t1}, \theta_0) \right)$$

$$\delta_{aV}(r_{t1}, \theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im\left(Z_a(r_{t1}, \theta_0) \right) \qquad \delta_{aV}(r_{t1}, \theta_0) = 2.284 \times 10^{-5}$$

$$\mu_{aV}(x, \theta_0) := -86400 \cdot \delta_{aV}(x \cdot r_{Ph}, \theta_0) \qquad \boxed{\mu_{aV}(x_1, \theta_0) = -1.974} \qquad \boxed{\mu_{aV}(x_1, 180 \cdot deg) = -0.962}$$

Courbes Phillips

$\mu_{aVPh}(\theta) := \mu_{aV}(1, \theta)$

$\theta_m := 100 \cdot deg, 105 \cdot deg .. 360 \cdot deg$